CS70: Discrete Math and Probability

Fan Ye

June 27, 2016

Today

More graphs

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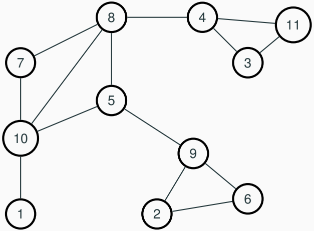
- Connectivity
- Eulerian Tour
- Planar graphs
  - 5 coloring theorem

Connectivity



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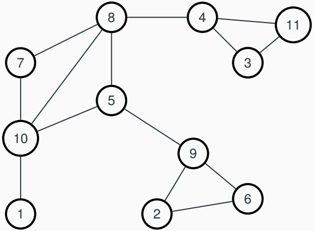

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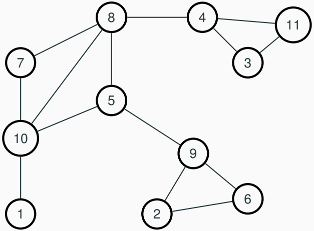

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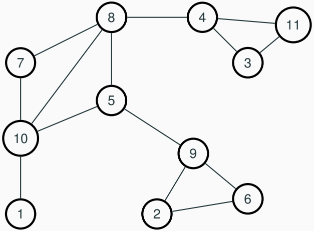

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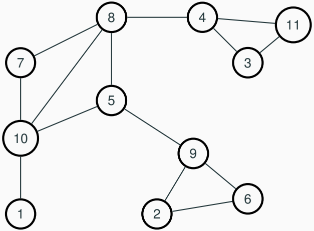

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Is graph connected? Yes?

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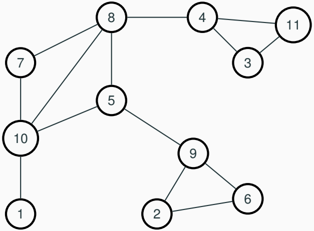

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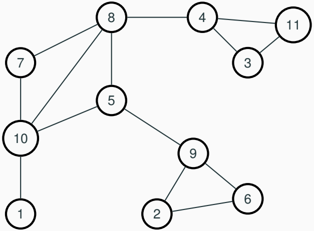

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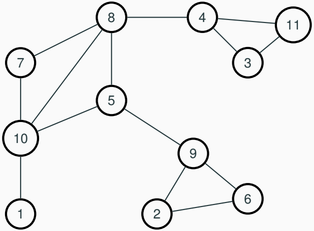

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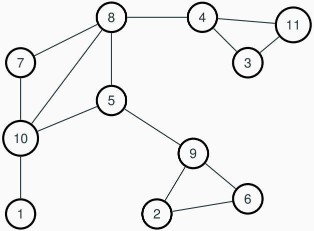

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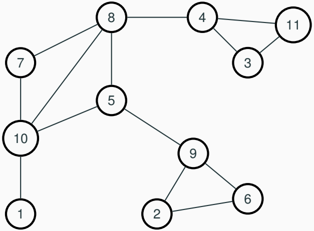

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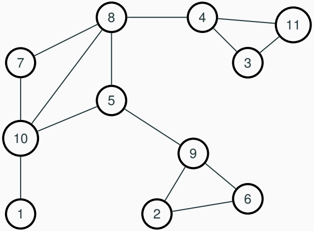

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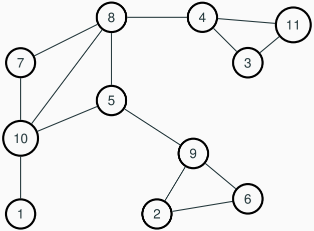

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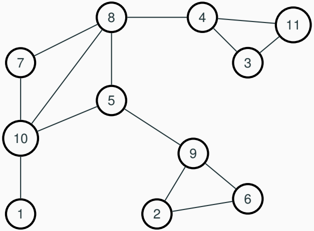

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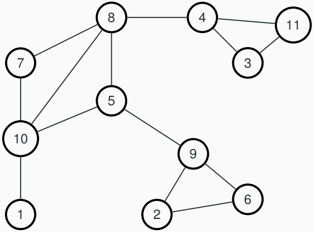
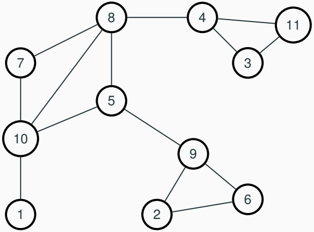


Is graph above connected?

Connected component



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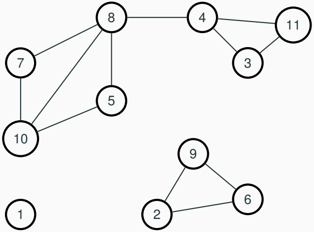
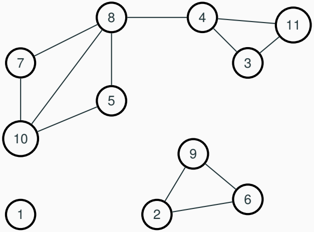


Is graph above connected? Yes!

How about now?

Connected component



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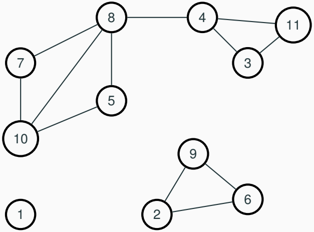

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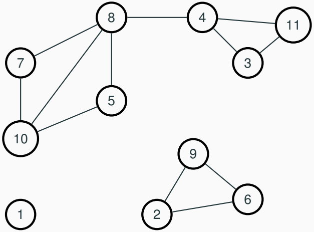
Connected Components?

Connected component

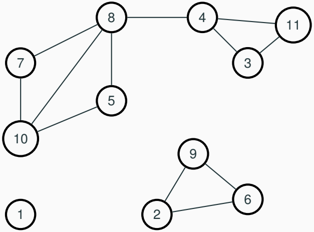


Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected component



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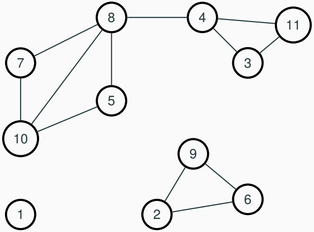
Connected component - maximal set of connected vertices.

Connected component



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Quick Check: Is $\{10,7,5\}$ a connected component?

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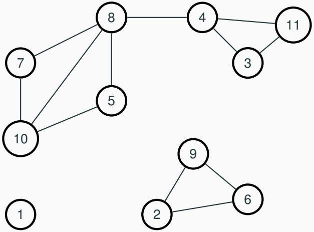

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Finally..back to bridges!

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Definition:

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Eulerian Tour is connected so graph is connected.

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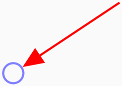

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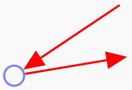

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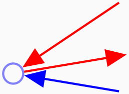

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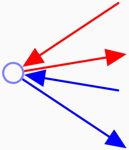

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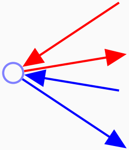

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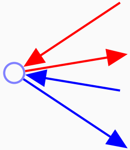

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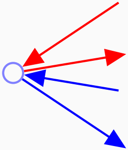

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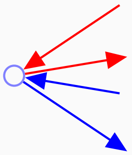

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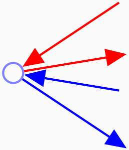

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Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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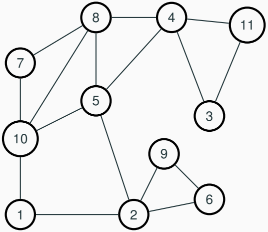

1. Take a walk starting from v (1) on "unused" edges

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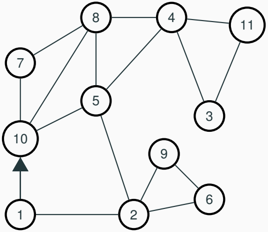

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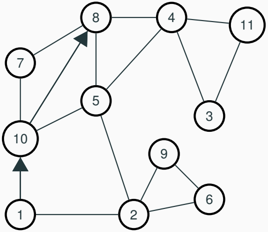

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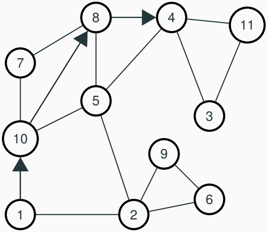

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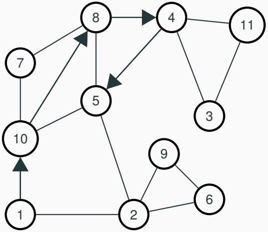

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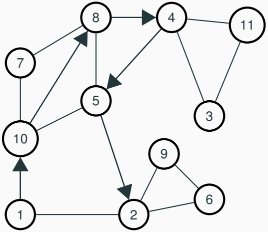

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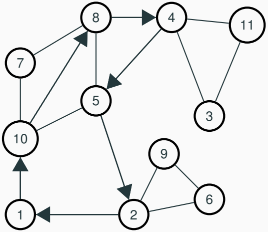

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... till you get back to v.

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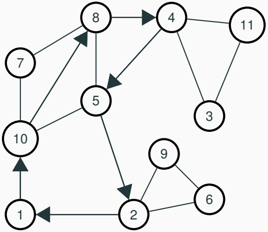

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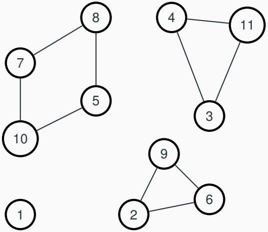
... till you get back to v.
2. Remove tour, C.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.



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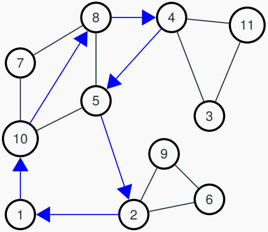

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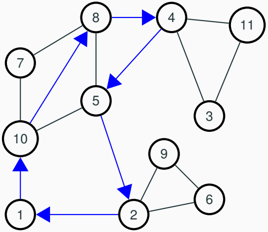

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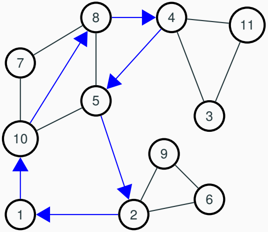

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3. Let $G_1, \ldots, G_k$ be connected components.
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      Why? G was connected.

Finding a tour!

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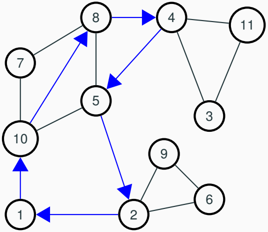

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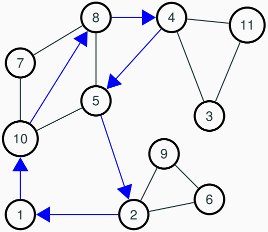

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   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
   Example: $v_1 = 1$,

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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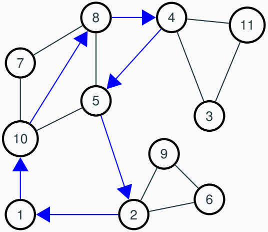

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3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$,

Finding a tour!

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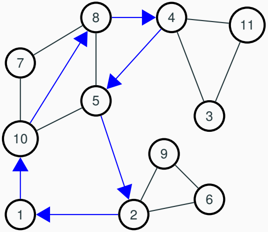

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3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

Finding a tour!

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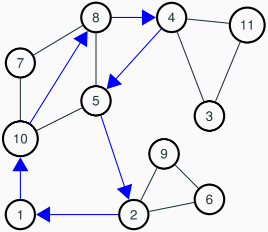

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      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.

Finding a tour!

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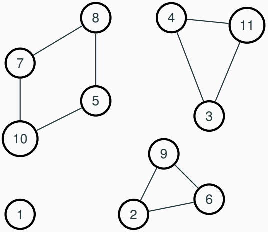

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4. Recurse on $G_1, \ldots, G_k$ starting from v_i

Finding a tour!

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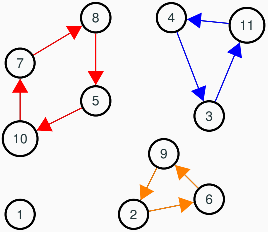

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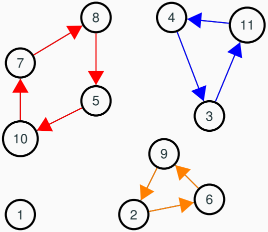

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   Why? G was connected.

   Let v_i be (first) node in G_i touched by C.

   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.

Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
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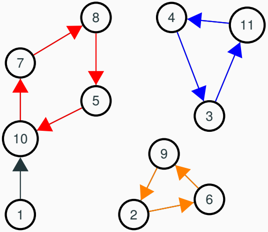

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5. Splice together.
   1,10

Finding a tour!

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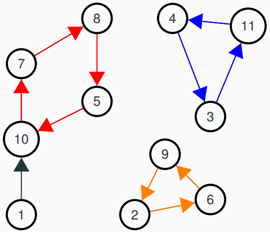

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4. Recurse on $G_1, \ldots, G_k$ starting from v_i
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   1,10,7,8,5,10

Finding a tour!

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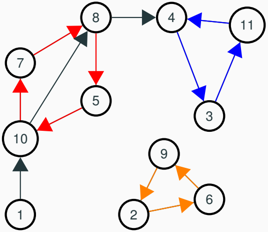

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   1,10,7,8,5,10 ,8,4

Finding a tour!

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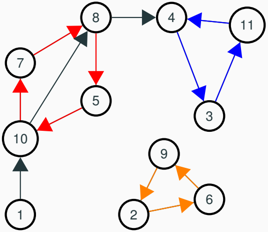

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   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
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5. Splice together.

   1,10,7,8,5,10 ,8,4,3,11,4

Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
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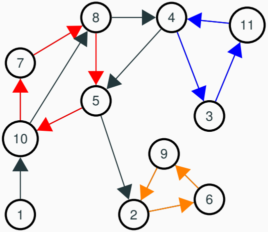

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2

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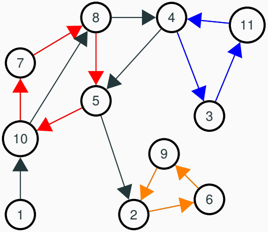

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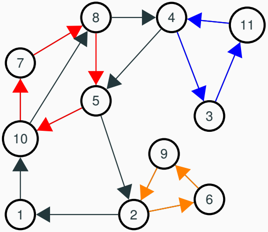

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Finding a tour: in general.

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Claim: Do get back to v!

Finding a tour: in general.

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Claim: Do get back to v!
Proof of Claim: Even degree.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
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2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

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Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
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Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\Longrightarrow$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
  Why is there a v_i in C?
    G was connected $\implies$
      a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
  Prf: Tour C has even incidences to any vertex v. □

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\Longrightarrow$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. □

3. Find tour T_i of G_i

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. □

3. Find tour T_i of G_i starting/ending at v_i.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
 Why is there a v_i in C?
  G was connected $\implies$
   a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
 Prf: Tour C has even incidences to any vertex v. $\square$

3. Find tour T_i of G_i starting/ending at v_i. Induction.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. $\square$

3. Find tour T_i of G_i starting/ending at v_i. Induction.
4. Splice T_i into C where v_i first appears in C.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
  Why is there a v_i in C?
    G was connected $\implies$
      a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
  Prf: Tour C has even incidences to any vertex v. □

3. Find tour T_i of G_i starting/ending at v_i. Induction.
4. Splice T_i into C where v_i first appears in C.

Visits every edge once:
  Visits edges in C

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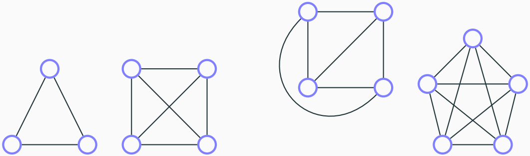
Planar graphs.

A graph that can be drawn in the plane without edge crossings.

Planar graphs.

A graph that can be drawn in the plane without edge crossings.



Planar?

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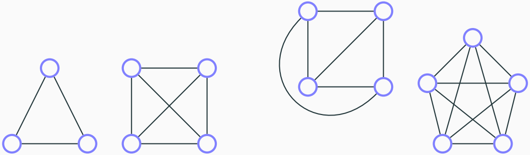

Planar? Yes for Triangle.

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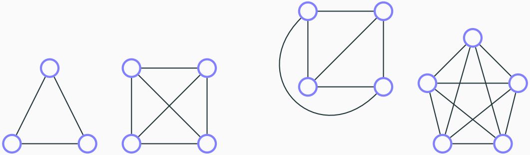

Planar? Yes for Triangle.
Four node complete?

Planar graphs.

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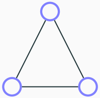

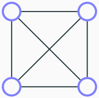

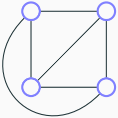

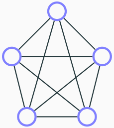

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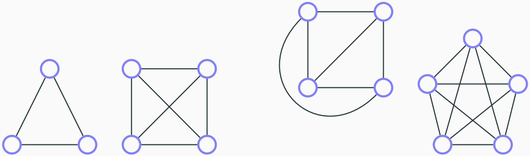

Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or K_5?

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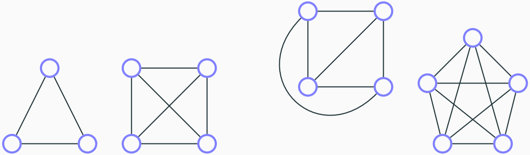

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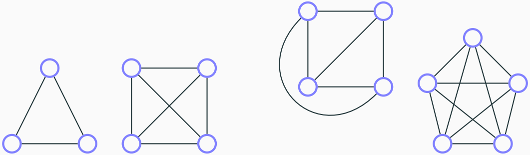

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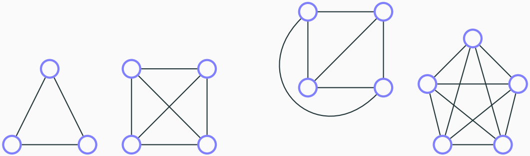

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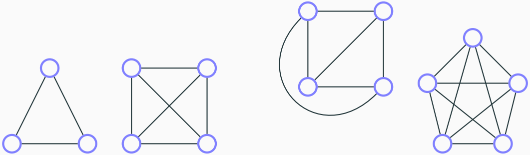

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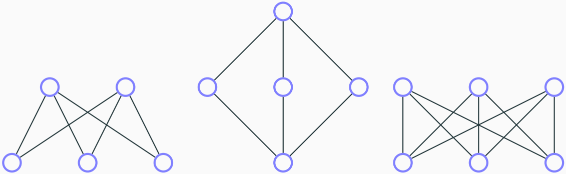

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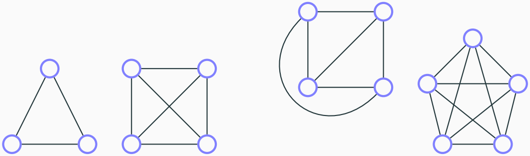

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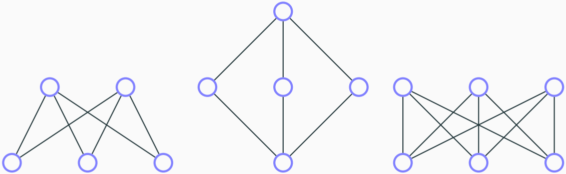

Two to three nodes, bipartite?

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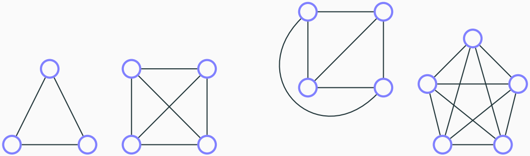

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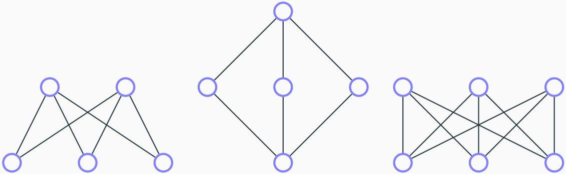

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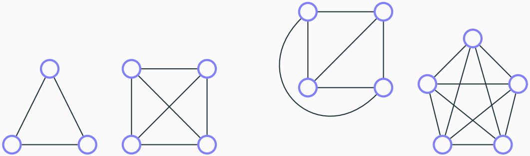

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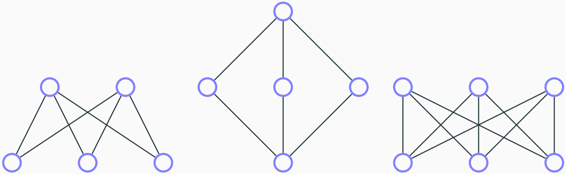

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Three to three nodes, complete/bipartite or $K_{3,3}$.

Planar graphs.

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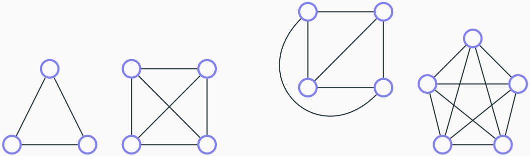

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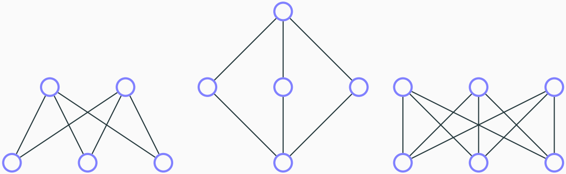

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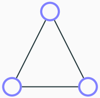
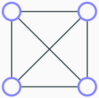
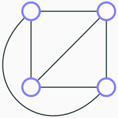
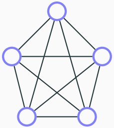

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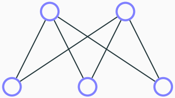
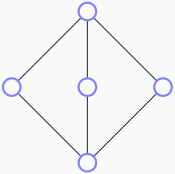
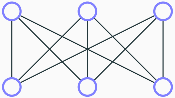

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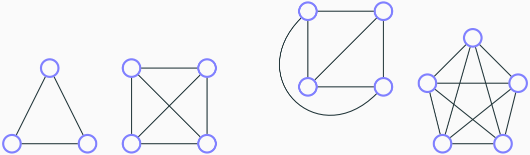

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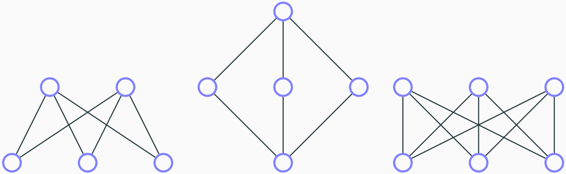

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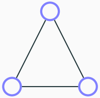
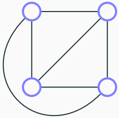
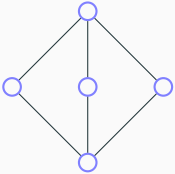
Euler's Formula.







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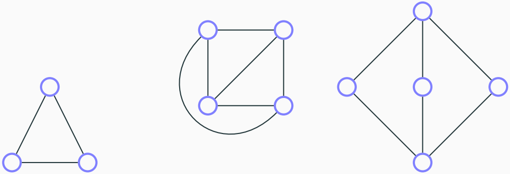
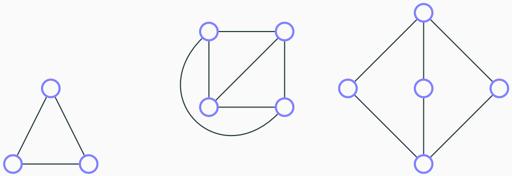


Faces: connected regions of the plane.

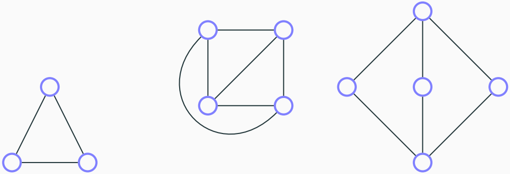
Euler's Formula.



Faces: connected regions of the plane.

How many faces for

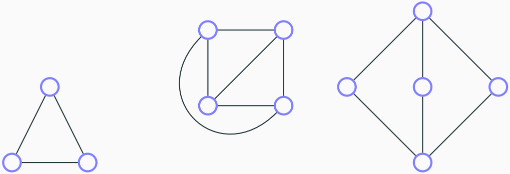
Euler's Formula.



Faces: connected regions of the plane.

How many faces for
triangle?

Euler's Formula.

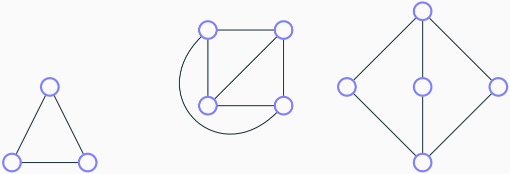


Faces: connected regions of the plane.

How many faces for

triangle? 2

Euler's Formula.



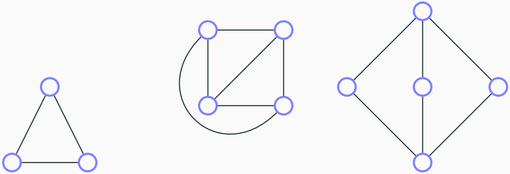
Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or K_4?

Euler's Formula.



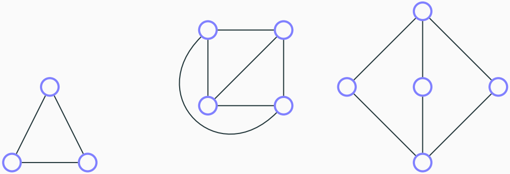
Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or K_4? 4

Euler's Formula.



Faces: connected regions of the plane.

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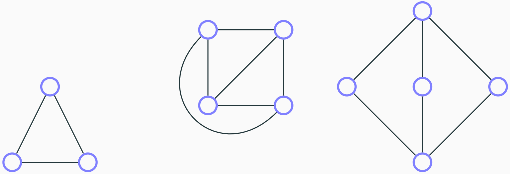
Euler's Formula.



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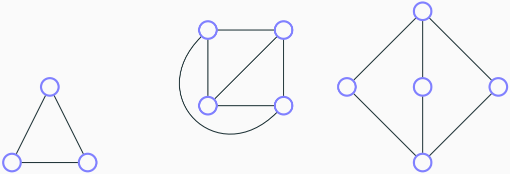

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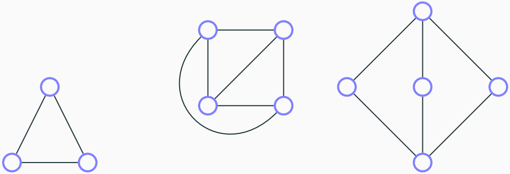

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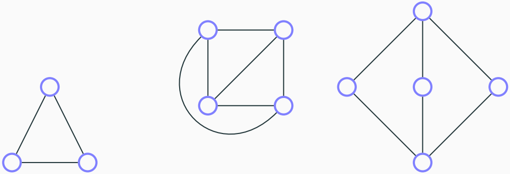
v is number of vertices, e is number of edges, f is number of faces.

Euler's Formula.



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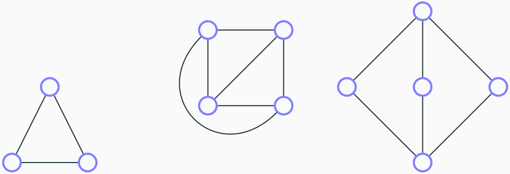

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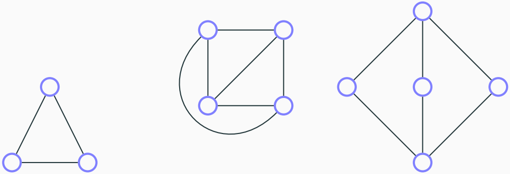

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Triangle:

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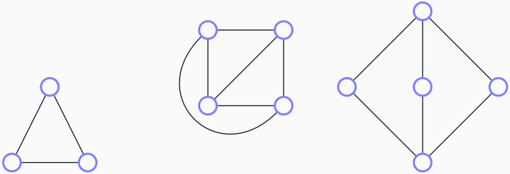

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Triangle: $3+2=3+2$!

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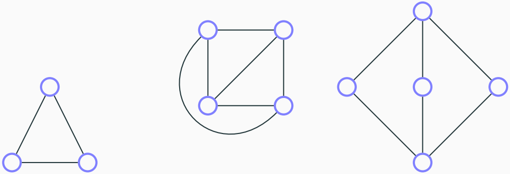

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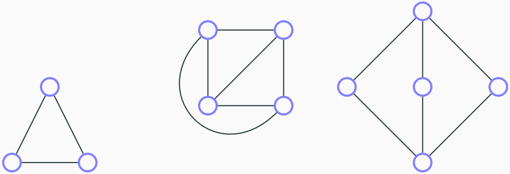
K_4:

Euler's Formula.



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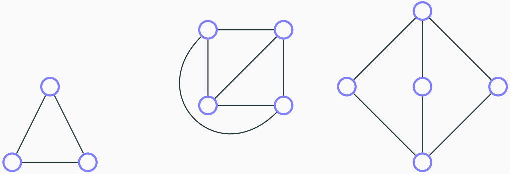
Triangle: $3 + 2 = 3 + 2$!
K_4: $4 + 4 = 6 + 2$!

Euler's Formula.



Faces: connected regions of the plane.

How many faces for
triangle? 2
complete on four vertices or K_4? 4
bipartite, complete two/three or $K_{2,3}$? 3

v is number of vertices, e is number of edges, f is number of faces.

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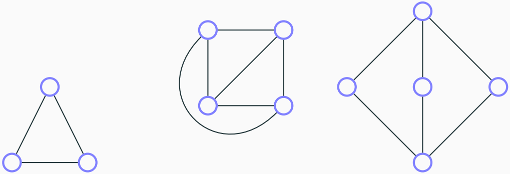

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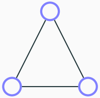

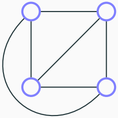

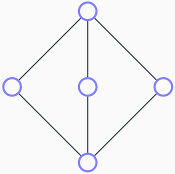

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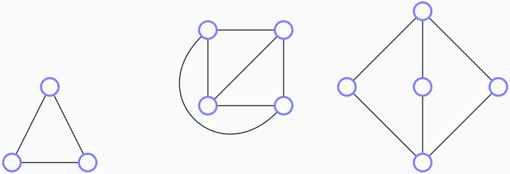

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Examples = 3!

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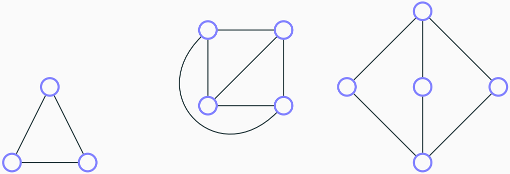

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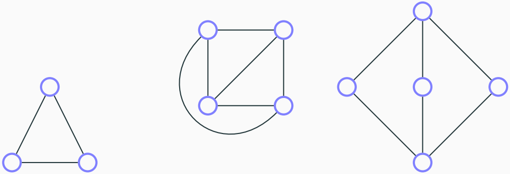

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Euler and Polyhedron.

Greeks knew formula for polyhedron.

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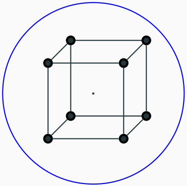

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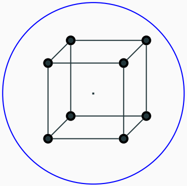

Faces?

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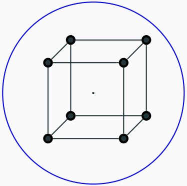

Faces? 6.   Edges?

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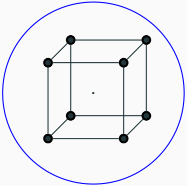

Faces? 6. Edges? 12.

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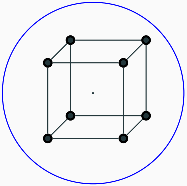

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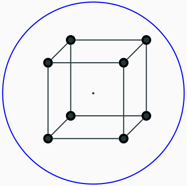

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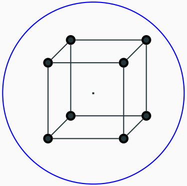

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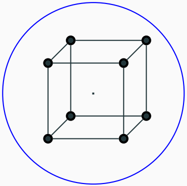

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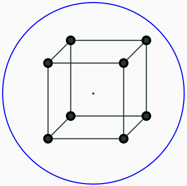

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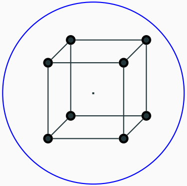

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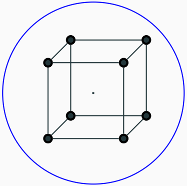

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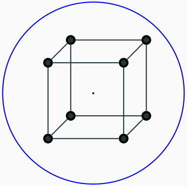

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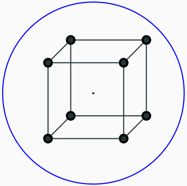

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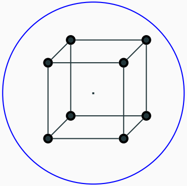

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Polyhedron without holes

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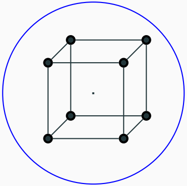

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Polyhedron without holes $\equiv$

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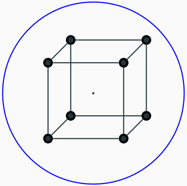

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Polyhedron without holes $\equiv$ Planar graphs.

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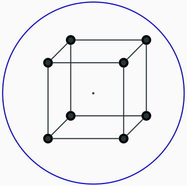

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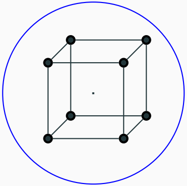
Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.

Euler and Polyhedron.

Greeks knew formula for polyhedron.



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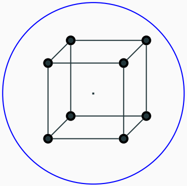
Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.

Euler and Polyhedron.

Greeks knew formula for polyhedron.



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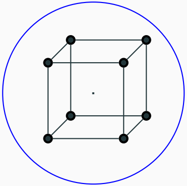
Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere

Euler and Polyhedron.

Greeks knew formula for polyhedron.



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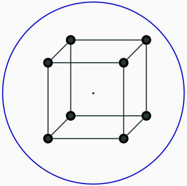
Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane!

Euler and Polyhedron.

Greeks knew formula for polyhedron.



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Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane! Topologically.

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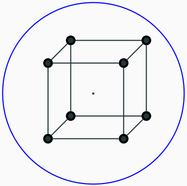

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Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
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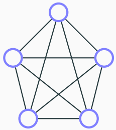
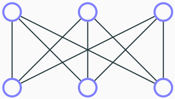
Sphere $\equiv$ Plane! Topologically.

Euler proved formula thousands of years later!

Euler and planarity of K_5 and $K_{3,3}$





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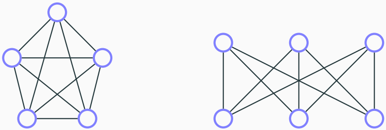

Euler: $v+f=e+2$ for connected planar graph.

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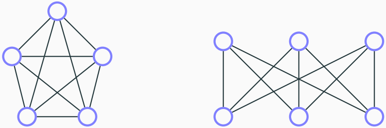

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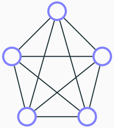
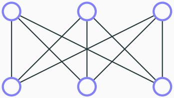
Each face is adjacent to at least three edges.

Euler and planarity of K_5 and $K_{3,3}$

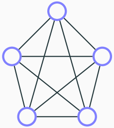


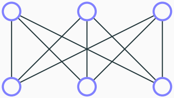


Euler: $v + f = e + 2$ for connected planar graph.

Each face is adjacent to at least three edges. face-edge adjacencies. $\geq 3f$

Euler and planarity of K_5 and $K_{3,3}$





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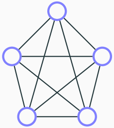
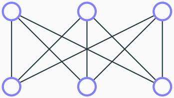
Each face is adjacent to at least three edges. face-edge adjacencies. $\geq 3f$
Each edge is adjacent to exactly two faces.

Euler and planarity of K_5 and $K_{3,3}$

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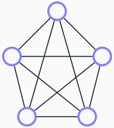

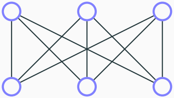

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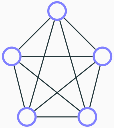

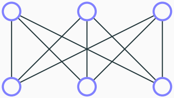

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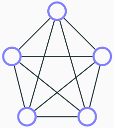

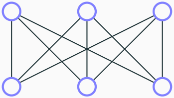

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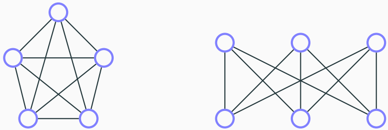
Euler and planarity of K_5 and $K_{3,3}$



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K_5

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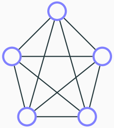

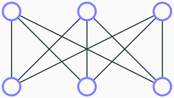

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K_5 Edges?

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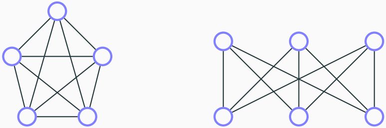

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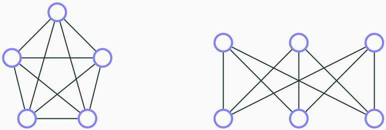

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K_5 Edges? $4 + 3 + 2 + 1 = 10$.

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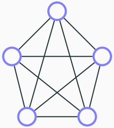

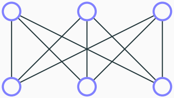

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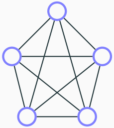

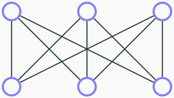

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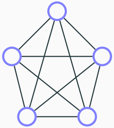

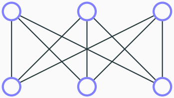

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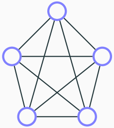

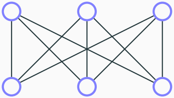

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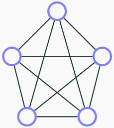
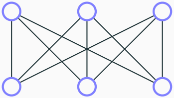

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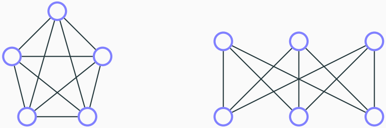

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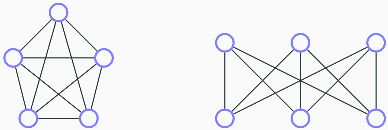
Euler and planarity of K_5 and $K_{3,3}$



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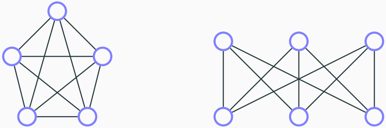

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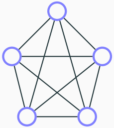

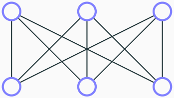

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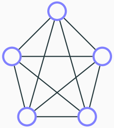

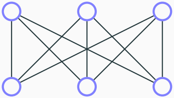

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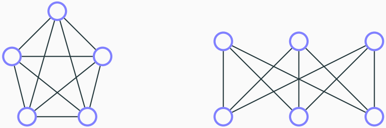

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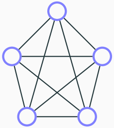

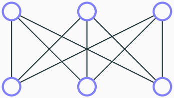

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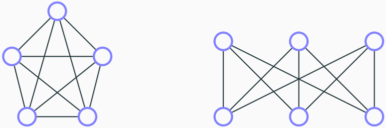

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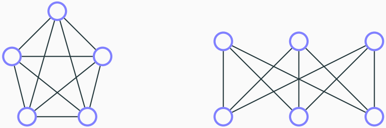

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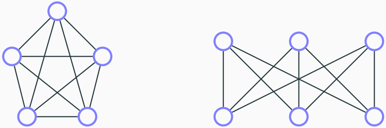

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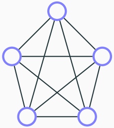

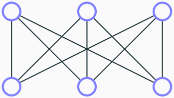

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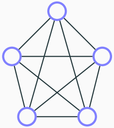

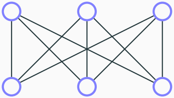

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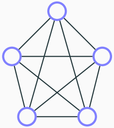

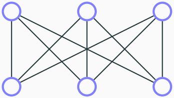

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Tree.

A tree is a connected acyclic graph.

Tree.

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To tree or not to tree!

Tree.

A tree is a connected acyclic graph.

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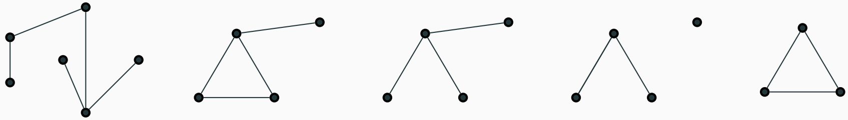

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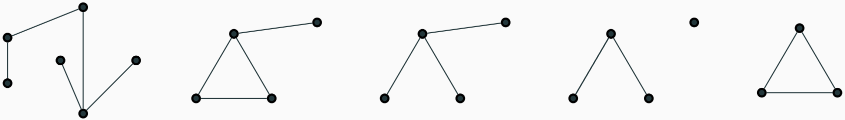

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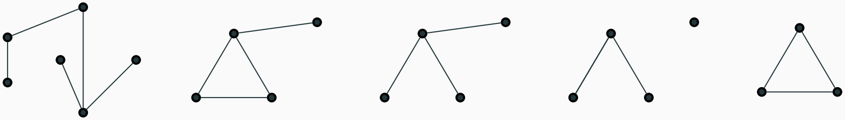

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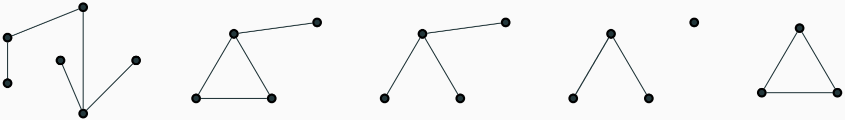

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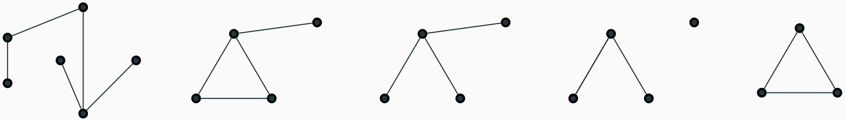

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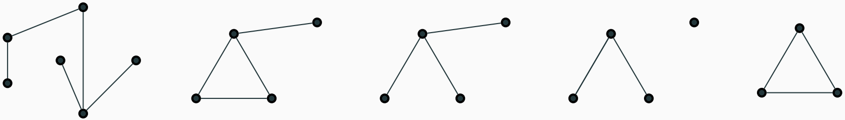

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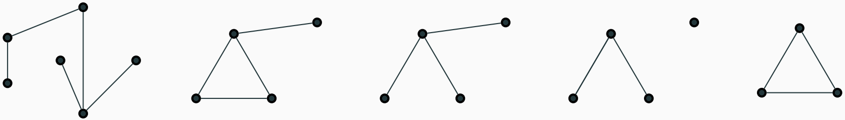

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Faces?

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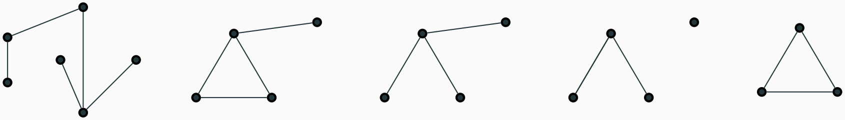

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Faces? 1.

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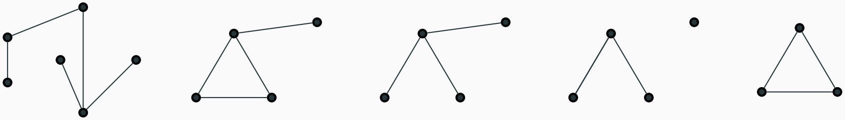

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Faces? 1. 2.

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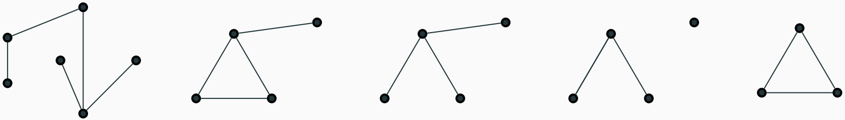

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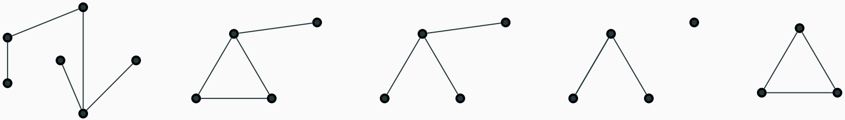

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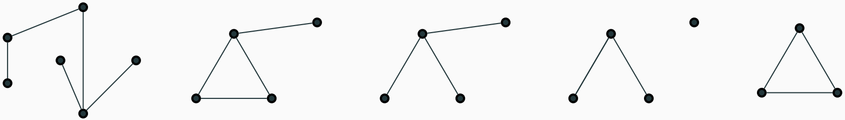

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Faces? 1. 2. 1. 1. 2.

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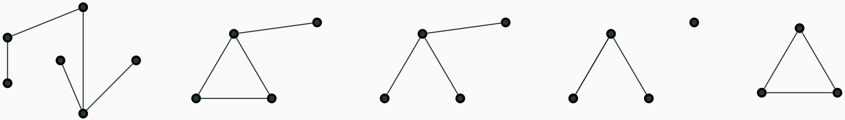

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Vertices/Edges.

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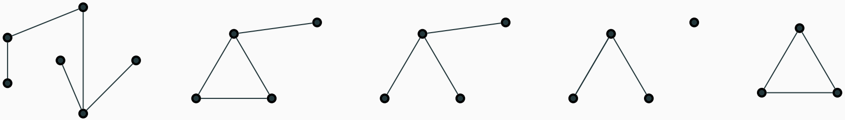

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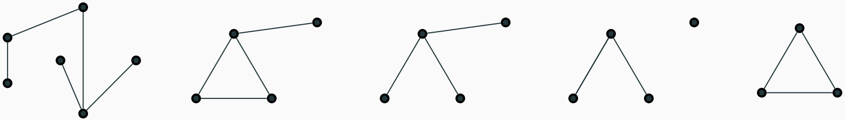

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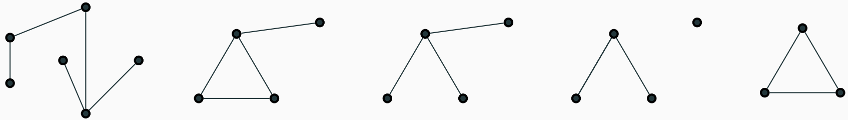

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One face for trees!

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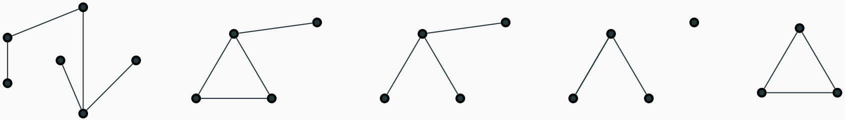

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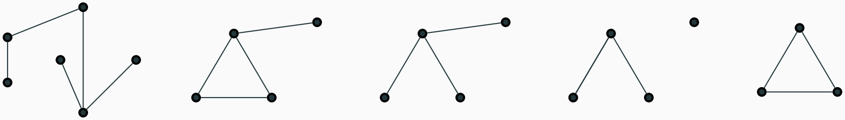

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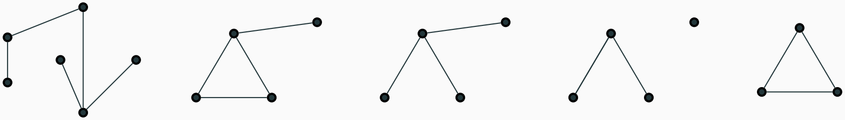

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Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

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Proof sketch:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof sketch: Induction on e.
Base:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$,

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$. $p(0)$ (base case) holds

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Induction Step:

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Induction Step:
If it is a tree.

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Induction Step:
  If it is a tree. Done.

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Base: $e = 0$, $v = f = 1$. $p(0)$ (base case) holds
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle.

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Base: $e = 0$, $v = f = 1$. $p(0)$ (base case) holds
Induction Step:
  If it is a tree. Done.
  If not a tree.
    Find a cycle. Remove edge.

Euler's formula.

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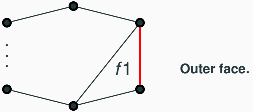

Joins two faces.

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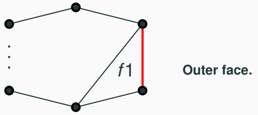

Joins two faces.
New graph: v-vertices.

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

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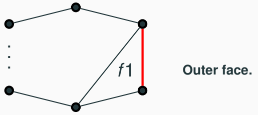

Joins two faces.
New graph: v-vertices. $e-1$ edges.

Euler's formula.

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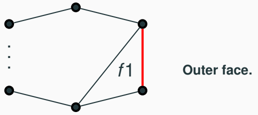

Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces.

Euler's formula.

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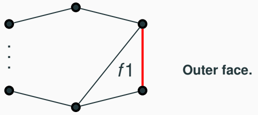

Joins two faces.
New graph: v-vertices. $e-1$ edges. $f-1$ faces. Planar.

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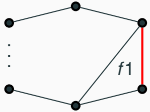

Outer face.

Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces. Planar.
$v + (f - 1) = (e - 1) + 2$ by induction hypothesis for a smaller graph with $e - 1$ edges.

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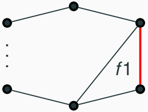

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$v+(f-1)=(e-1)+2$ by induction hypothesis for a smaller graph with $e-1$ edges.
Therefore $v+f=e+2$.

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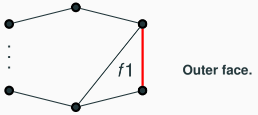

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Given $G = (V, E)$, a coloring of a G assigns colors to vertices V where for each edge the endpoints have different colors.

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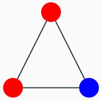

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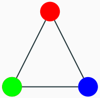

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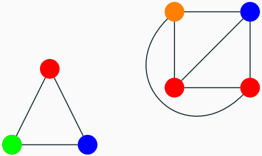

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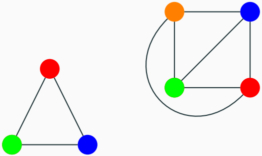

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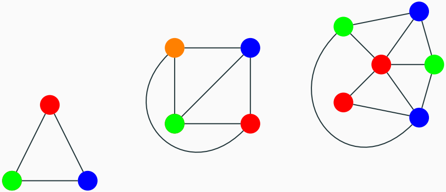

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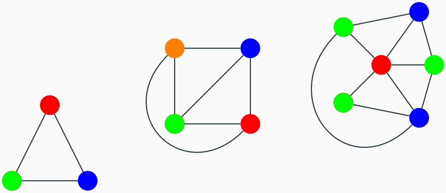

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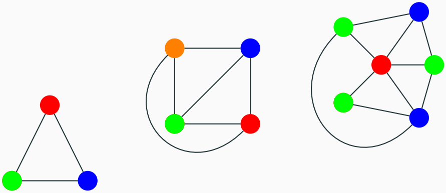

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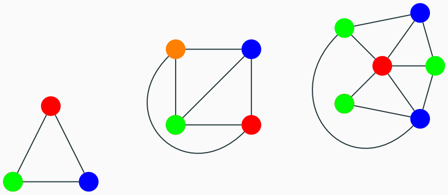

Notice that the last one, has one three colors.

Graph Coloring.

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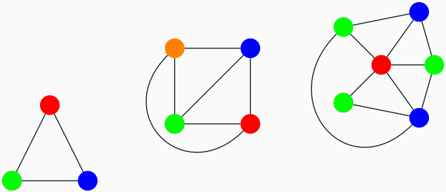

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Fewer colors than number of vertices.

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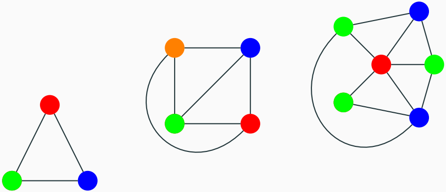

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Fewer colors than max degree node.

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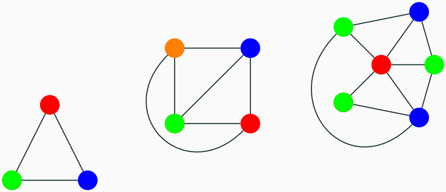

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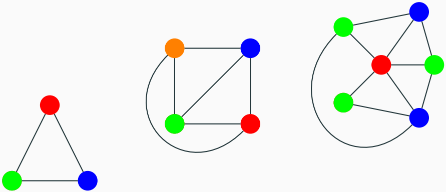

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Interesting things to do.

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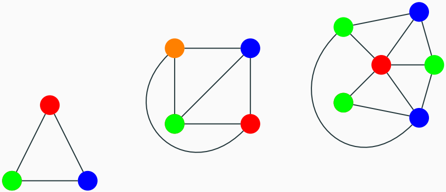

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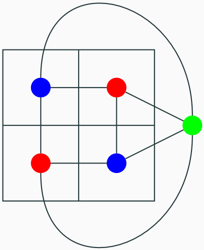
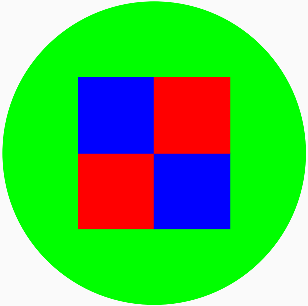
Interesting things to do. Algorithm!

Planar graphs and maps.

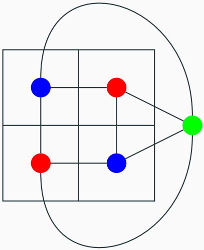
Planar graph coloring $\equiv$ map coloring.

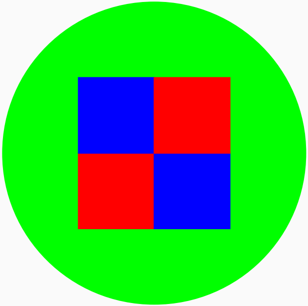




Planar graphs and maps.

Planar graph coloring $\equiv$ map coloring.





Four color theorem is about planar graphs!

Six color theorem.

Theorem: Every planar graph can be colored with six colors.

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Proof:

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Total degree: $2e$

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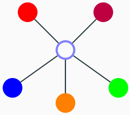

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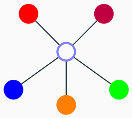

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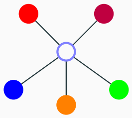

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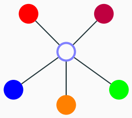

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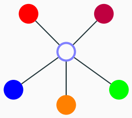

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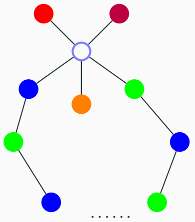

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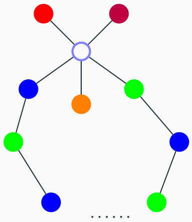

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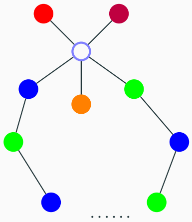

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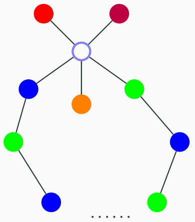

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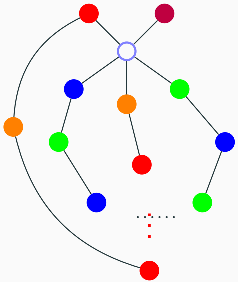

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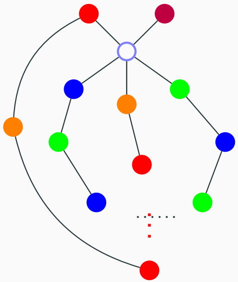

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Planar.

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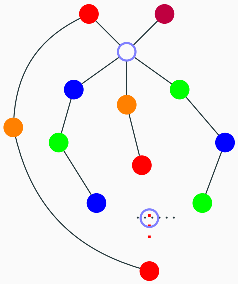

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Planar. $\implies$ paths intersect at a vertex!

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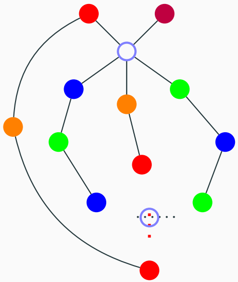

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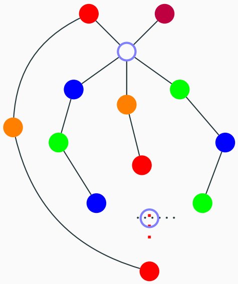

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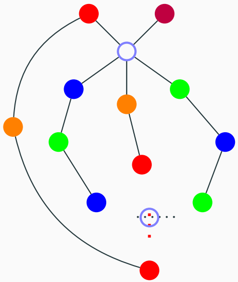

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Must be blue or green to be on that path.

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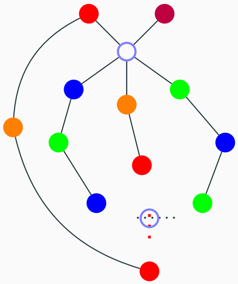

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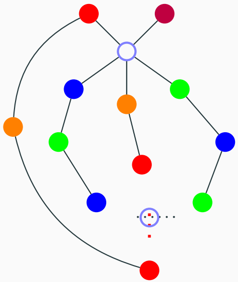

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Contradiction.

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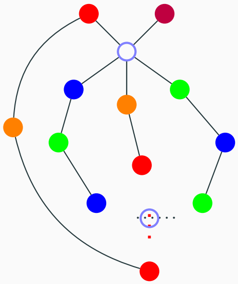

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Can recolor one of the neighbors.
And recolor "center" vertex.

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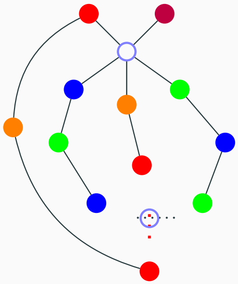

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