CS70: Discrete Math and Probability

June 21, 2016

Lecture 2: Proofs!

1. Direct proof
2. by Contraposition
3. by Contradiction
4. by Cases

Quick Background and Notation.

Integers closed under addition.

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

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Add $99a+11b$ to both sides.

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Assumed P: $11|a-b+c$. Proved Q: $11|n$.

The Converse

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Proof by contradiction:form

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Contrapositive: True $\implies P$.

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Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

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Did we prove?

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- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
-

$$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}}$$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

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Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.
- $$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}} = \sqrt{2}^2$$

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Thus, we have irrational x and y with a rational x^y (i.e., 2).

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One of the cases is true so theorem holds. □

Question: Which case holds?

Proof by cases.

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Thus, we have irrational x and y with a rational x^y (i.e., 2).

One of the cases is true so theorem holds. □

Question: Which case holds? Don't know!!!

Be careful.

Theorem: $3 = 4$

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Be careful.

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Proof: Assume $3 = 4$.

Start with $12 = 12$.

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Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$4 = 3$.

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By commutativity

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Divide one side by 3 and the other by 4 to get
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By commutativity theorem holds. $\square$

Be careful.

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Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$4 = 3$.

By commutativity theorem holds. □

Don't assume what you want to prove!

Be really careful!

Theorem: $1 = 2$
Proof:

Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$(x^2 - xy) = x^2 - y^2$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$(x^2 - xy) = x^2 - y^2$$
$$x(x - y) = (x + y)(x - y)$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$\begin{aligned}(x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y)\end{aligned}$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$\begin{aligned}(x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y) \\ x &= 2x\end{aligned}$$

Be really careful!

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$$
\begin{aligned}
(x^2 - xy) &= x^2 - y^2 \\
x(x-y) &= (x+y)(x-y) \\
x &= (x+y) \\
x &= 2x \\
\\
1 &= 2
\end{aligned}
$$

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Dividing by zero is no good.

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Also: Multiplying inequalities by a negative.

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Also: Multiplying inequalities by a negative.

$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

Direct Proof:

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Direct Proof:

To Prove: $P \implies Q$.

Summary: Note 2.

Direct Proof:

To Prove: $P \implies Q$. Assume P.

Summary: Note 2.

Direct Proof:

To Prove: $P \implies Q$. Assume P. Prove Q.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

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To Prove: $P \implies Q$

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$.

Summary: Note 2.

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To Prove: $P \implies Q$. Assume P. Prove Q.

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Summary: Note 2.

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To Prove: P

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By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

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Universal: show that statement holds in all cases.

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Existence: used cases where one is true.

Summary: Note 2.

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Careful when proving!

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Careful when proving!
Don't assume the theorem.

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To Prove: $P \implies Q$. Assume P. Prove Q.

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Don't assume the theorem. Divide by zero.

Summary: Note 2.

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To Prove: $P \implies Q$. Assume P. Prove Q.

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To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

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Careful when proving!
Don't assume the theorem. Divide by zero.Watch converse.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

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To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

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To Prove: P Assume $\neg P$. Prove False .

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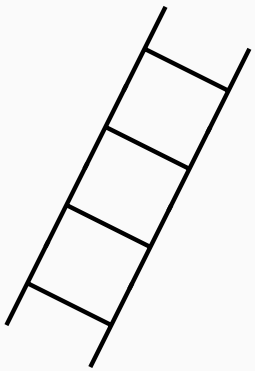
Don't assume the theorem. Divide by zero.Watch converse. ...

CS70: Note 3. Induction!

1. The natural numbers.
2. 5 year old Gauss.
3. ..and Induction.
4. Simple Proof.

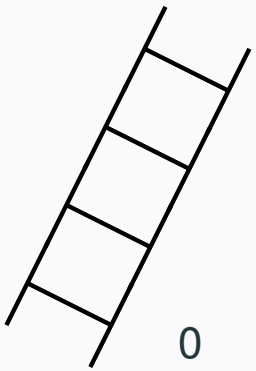
The naturals.

The naturals.



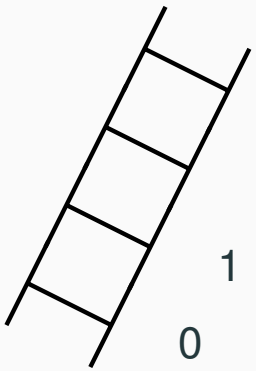
The naturals.

0,



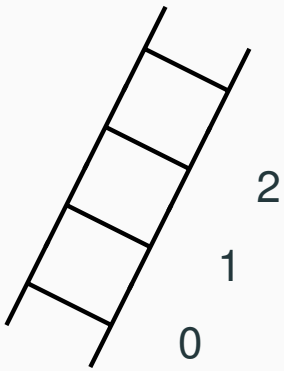
The naturals.

0, 1,



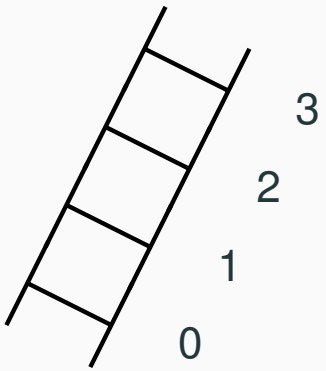
The naturals.

0, 1, 2,

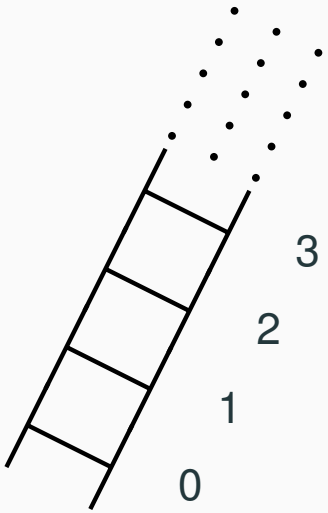


The naturals.

0, 1, 2, 3,



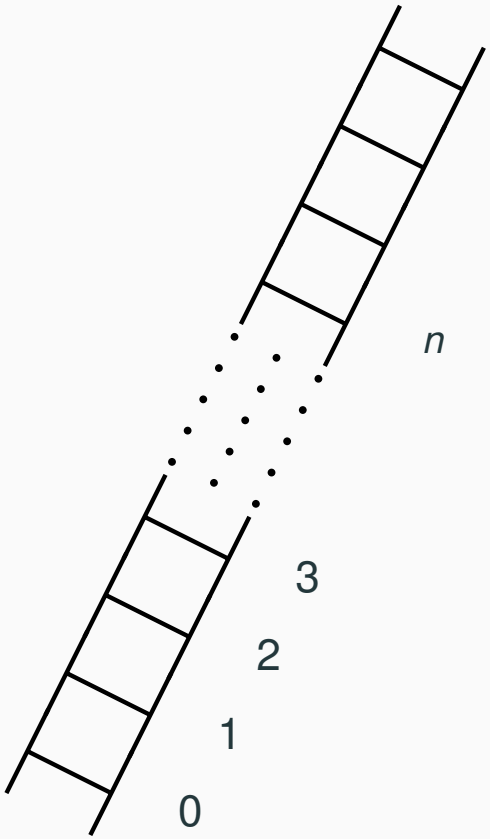
The naturals.



0, 1, 2, 3,

. . .,

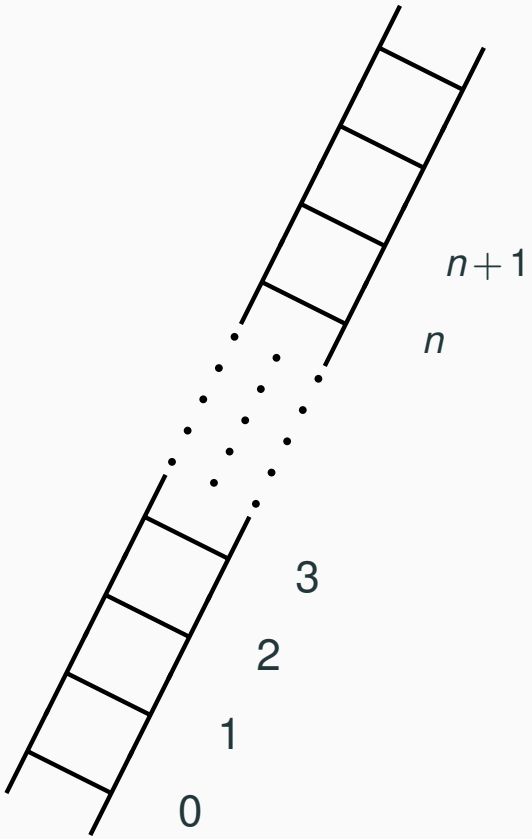
The naturals.



0, 1, 2, 3,

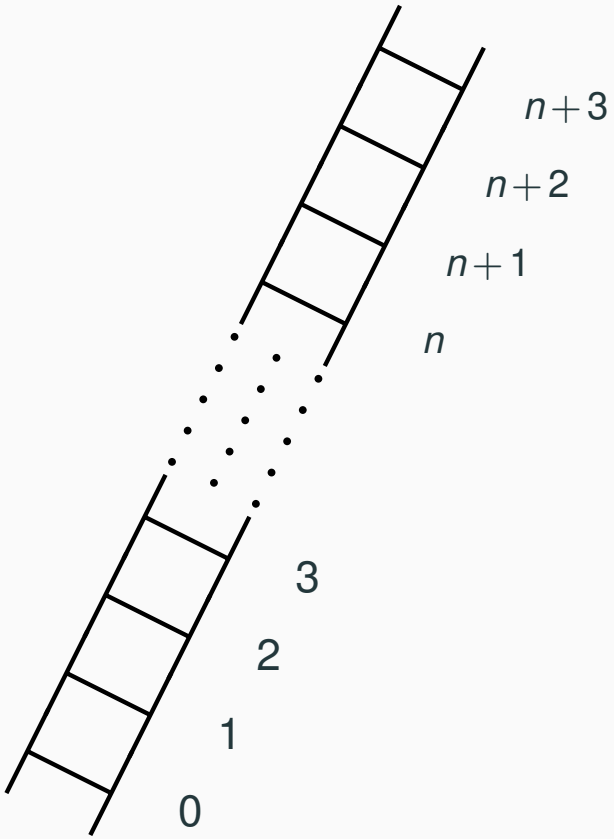
…, n,

The naturals.



0, 1, 2, 3,
…, n, $n+1$,

The naturals.



0, 1, 2, 3,
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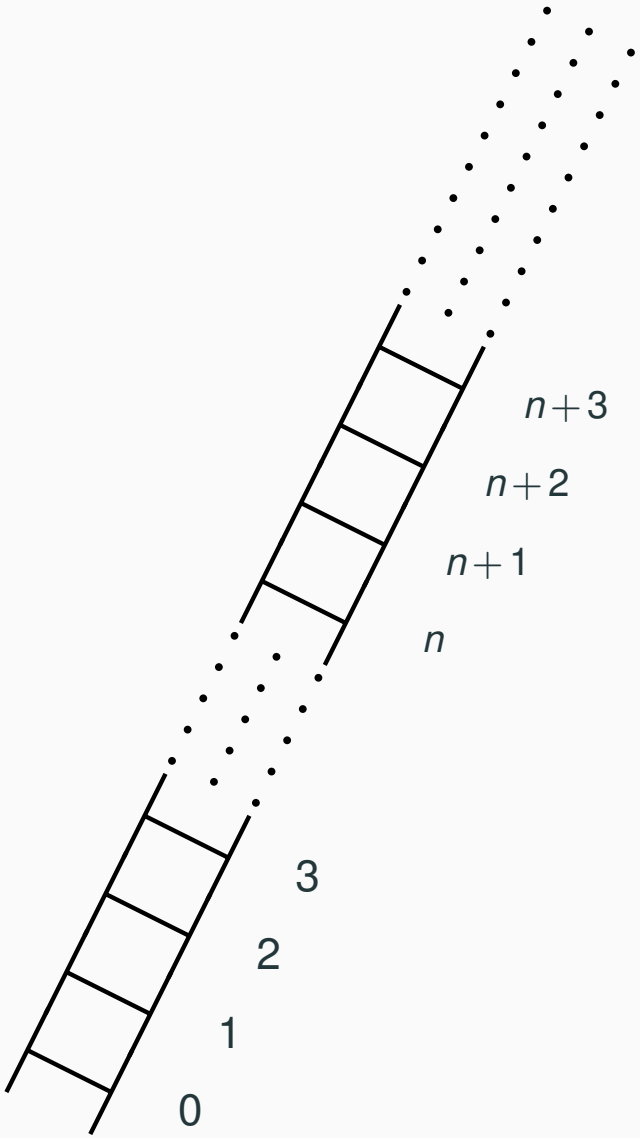

0, 1, 2, 3,

$\ldots, n, n+1, n+2, n+3, \ldots$

A formula.

A formula.

Teacher: Hello class.

A formula.

Teacher: Hello class.
Teacher:

A formula.

Teacher: Hello class.

Teacher: Please add the numbers from 1 to 100.

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's

A formula.

Teacher: Hello class.

Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$

A formula.

Teacher: Hello class.
Teacher: Please add the numbers from 1 to 100.

Gauss: It's $\frac{(100)(101)}{2}$ or 5050!

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Gauss and Induction

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Idea: assume predicate $P(n)$ for $n = k$.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Gauss and Induction

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Is predicate, $P(n)$ true for $n = k+1$?

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$\sum_{i=1}^{k+1} i$

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$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1)$

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$$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}.$$

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How about $k+2$.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

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Need to start somewhere.

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Statement is true for $n = 0$

Gauss and Induction

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Gauss and Induction

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plus inductive step

Gauss and Induction

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plus inductive step $\implies$ true for $n = 1$

Gauss and Induction

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plus inductive step $\implies$ true for $n = 2$

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Predicate, $P(n)$, True for all natural numbers!

Gauss and Induction

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