Today

Review for Midterm.

First there was logic...

A statement is a true or false.

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A statement is a true or false.
Don't worry about Gödel.

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Statements?

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Statements?
$3 = 4 - 1$?

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Statements?
$3 = 4 - 1$? Statement!

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Statements?
$3 = 4 - 1$? Statement!
$3 = 5$?

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$n = 3$?

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Predicate: Statement with free variable(s).

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- $3 = 4 - 1$? Statement!
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Predicate: Statement with free variable(s).

- Example: $x = 3$

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- Example: $x = 3$ Given a value for x, becomes a statement.

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Example: $x = 3$ Given a value for x, becomes a statement.
Predicate?
$n > 3$? Predicate: $P(n)$!

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Predicate?
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$x = y$?

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$n > 3$? Predicate: $P(n)$!

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- $x + y$? No.

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$x + y$? No. An expression, not a statement.

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Quantifiers:

$(\forall x)\ P(x)$.

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Quantifiers:
$(\forall x)\ P(x)$. For every x, $P(x)$ is true.

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$(\forall n \in N), n^2 \geq n$:

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$(\forall n \in N), n^2 \geq n$: Any free variables?

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$(\forall n \in N), n^2 \geq n$: Any free variables? No. So it's a statement.
$(\forall x \in R)(\exists y \in R) y > x$.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

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Propositional Expressions and Logical Equivalence

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
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Proofs: truth table or manipulation of known formulas.

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$$(A \implies B) \equiv (\neg A \vee B)$$
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$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

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Step 1:

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Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

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Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

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Or manipulate the formulas.

If you think it's not true:

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If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Find an example of $P(x)$ and $Q(x)$

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Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Find an example of $P(x)$ and $Q(x)$ such that one of the above steps fails.

...and then proofs...

Direct: $P \implies Q$

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Direct: $P \implies Q$
Example: a is even $\implies$ a^2 is even.

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Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even?

...and then proofs...

Direct: $P \implies Q$

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Approach: What is even? $a = 2k$

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$a^2 = 4k^2$

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Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

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$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication!

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication! So $2k^2$ is even.

...and then proofs...

Direct: $P \implies Q$

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Contrapositive: $P \implies Q$

...and then proofs...

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Integers closed under multiplication! So $2k^2$ is even.

a^2 is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

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Integers closed under multiplication! So $2k^2$ is even.

a^2 is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

Example: a^2 is odd $\implies a$ is odd.

...and then proofs...

Direct: $P \implies Q$

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Integers closed under multiplication! So $2k^2$ is even.
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Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.
Example: a^2 is odd $\implies a$ is odd.
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Example: rogue couple does not exist.

...jumping forward..

Contradiction in induction:

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Find a place where induction step doesn’t hold.

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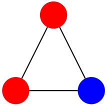

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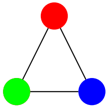

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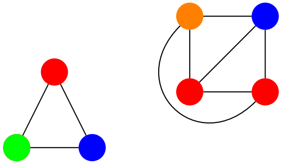

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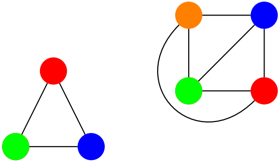

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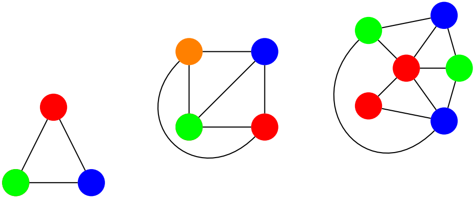

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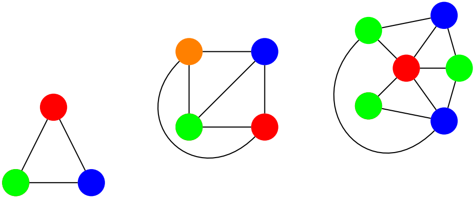

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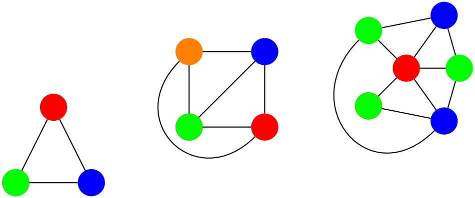

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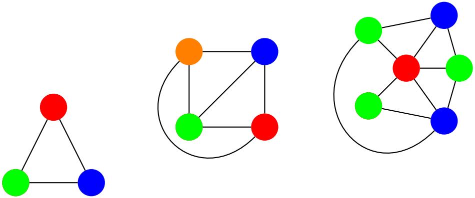

Notice that the last one, has one three colors.

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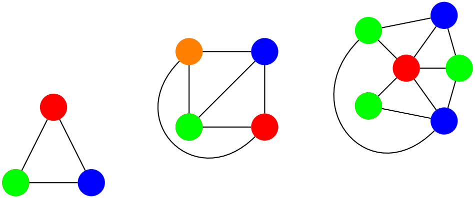

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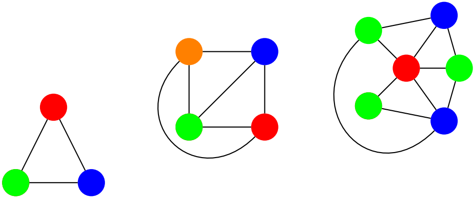

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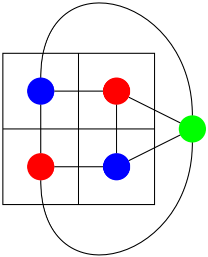
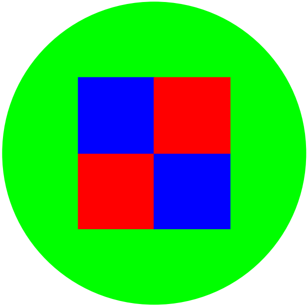
Fewer colors than number of vertices.

Fewer colors than max degree node.

Planar graphs and maps.

Planar graph coloring $\equiv$ map coloring.





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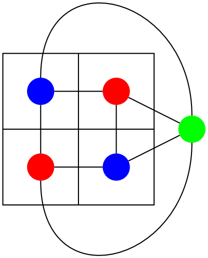

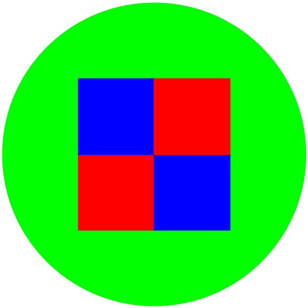

Four color theorem is about planar graphs!

Six color theorem.

Theorem: Every planar graph can be colored with six colors.

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Proof:

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Proof: Not Today!

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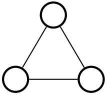
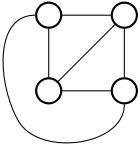
Graph Types: Complete Graph.





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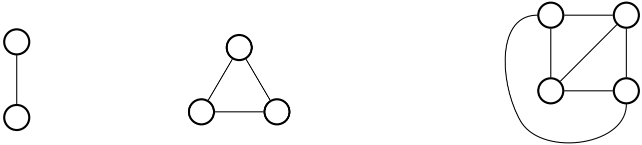

K_n, $|V| = n$

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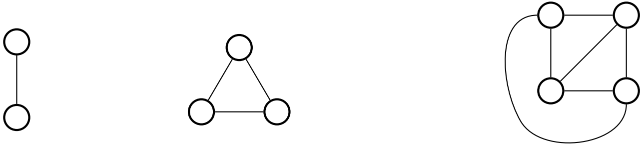

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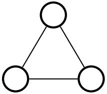

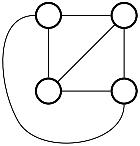

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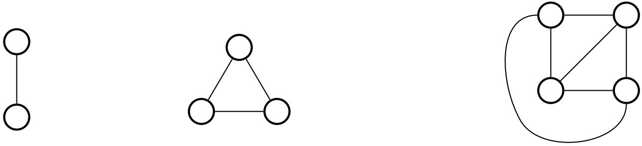

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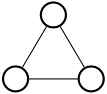
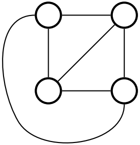

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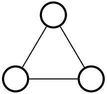

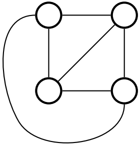

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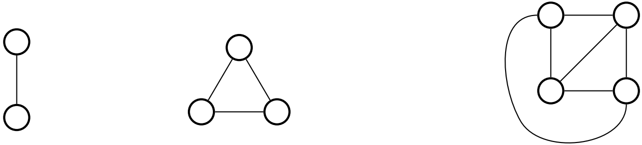

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Lots of edges: $n(n-1)/2$.

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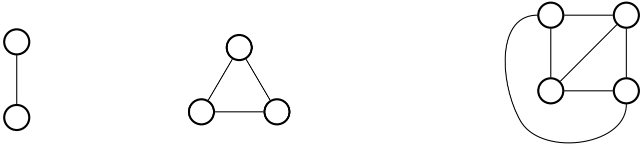

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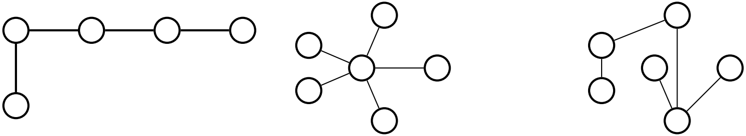
Trees.



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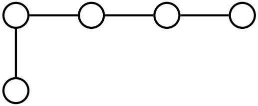 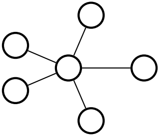 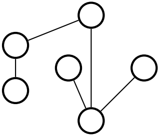

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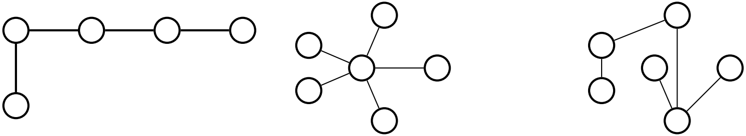

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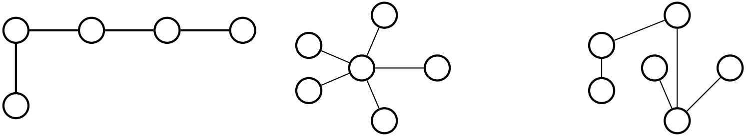

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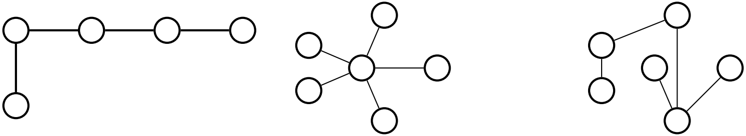

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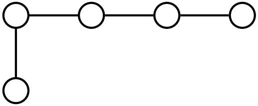

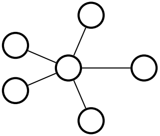

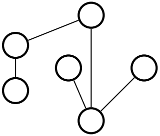

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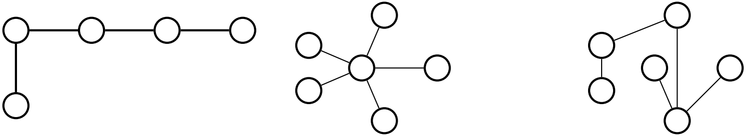

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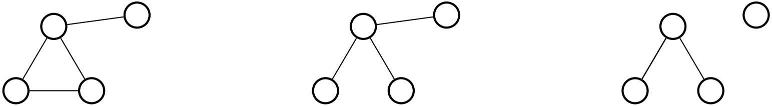
To tree or not to tree!



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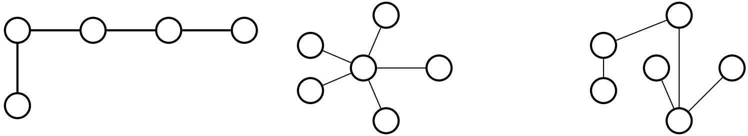

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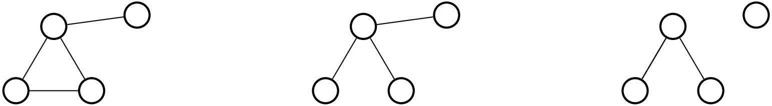
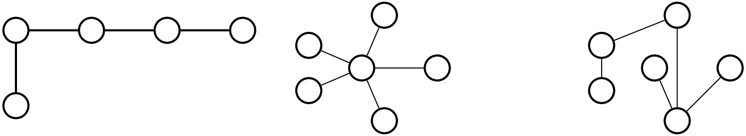


Minimally connected, minimum number of edges to connect.

Trees.



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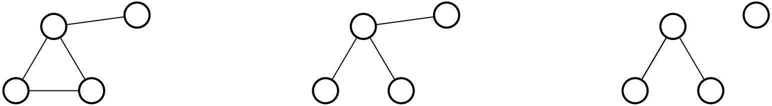

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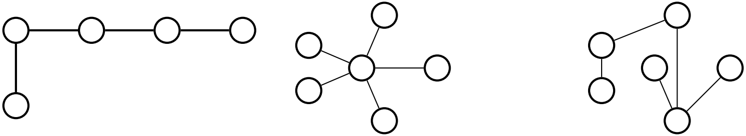

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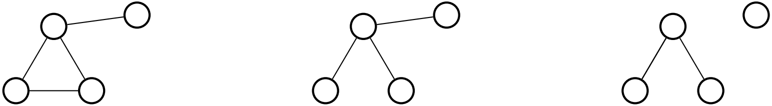

Minimally connected, minimum number of edges to connect.

Property:
Can remove a single node and break into components of size at most $|V|/2$.

Hypercube

Hypercubes.

Hypercube

Hypercubes. Really connected.

Hypercube

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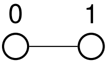

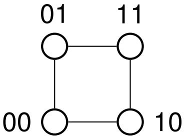

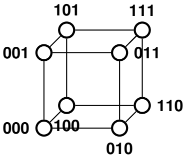

Recursive Definition.

A 0-dimensional hypercube is a node labelled with the empty string of bits.

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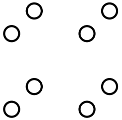

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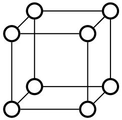

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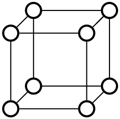

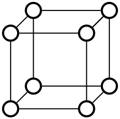

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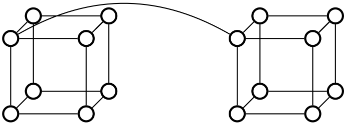

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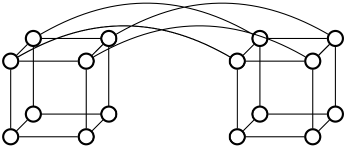

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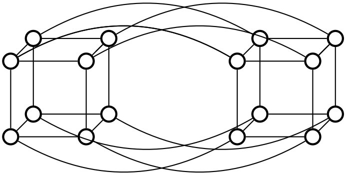

Hypercube:properties

Dense cuts: Cutting off k nodes needs $\geq k$ edges.

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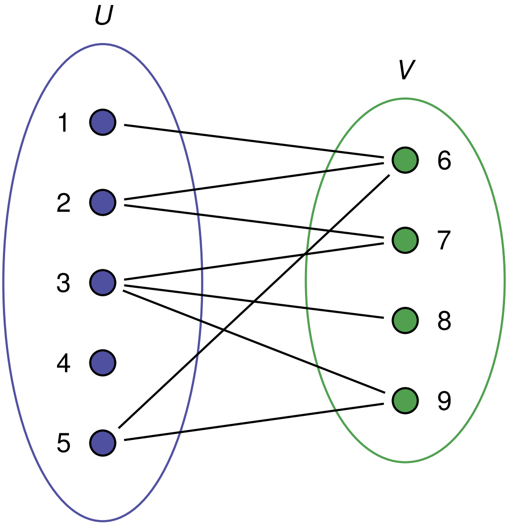
Good communication network!

Bipartite graphs



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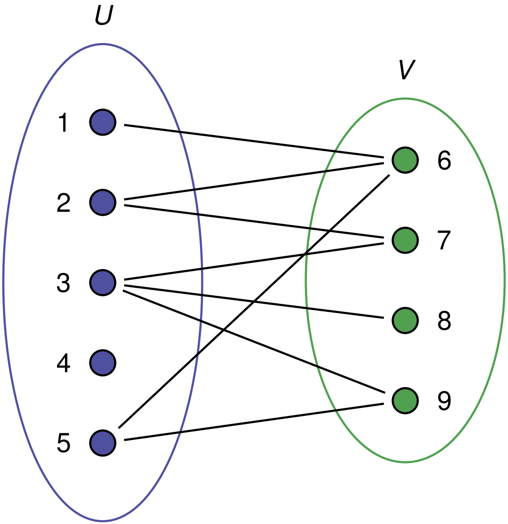

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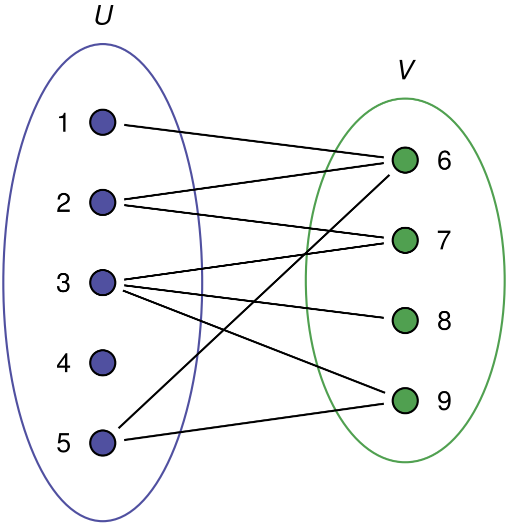

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Cycles have length 4 or more edges.

Stable Marriage: a study in definitions and WOP.

n-men, *n*-women.

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Does stable pairing exist?

Yes for matching.
No, for roommates problem.

TMA.

Traditional Marriage Algorithm:

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Every man proposes to his favorite woman from the ones that haven't already rejected him.

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Not rogue couple!

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Man optimal $\implies$ Woman pessimal.

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Man optimal $\implies$ Woman pessimal.
Woman optimal $\implies$ Man pessimal.

And then countability

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More than one infinities

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More than one infinities

Some things are countable

And then countability

More than one infinities

Some things are countable , like the natural numbers

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Why? **Diagonalization:** Well, assume there is a list. Can construct a diagonal element x. x is not in the list! Contradiction.

HALTING

HALTING

The HALT problem:

HALTING

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Like: Will this program P even print "Hello World"?

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The HALT problem: Is there a program that can tell you if another (generic) program halts on an input?

NO!

Why? Self reference!

Who cares? Using the same trick I can show that a bunch of problems are undecidable!
Like: Will this program P even print ”Hello World”?
Or ”Is there an input for this program P that will give an attacker admin access?

Counting!

Sample k items out of n.

	With Replacement	Without Replacement
Order matters	n^k	$\frac{n!}{(n-k)!}$
Order doesn't matter	$\binom{n+k-1}{n-1}$	$\binom{n}{k}$

Stars and bars!

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Confusion yesterday: 10 hats.

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Confusion yesterday: 10 hats. 7 days.

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Confusion yesterday: 10 hats. 7 days. I can wear the same hat on different days (replacement).

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How many stars? One for each day.

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How many bars? One fewer than the hats.

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Didn't wear hats 1 and 2. Wore hat 3 for 1 day, hat 4 for 2 days, hat 5 days. Didn't wear hats 6 and 7. Hat 8 for 3 days. Didn't wear hats 9 and 10.

Combinatorial Proofs.

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Add them up. (**Sum rule**)

Midterm format

Time: 110 minutes.

Midterm format

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Some short answers.

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Some short answers.
  Get at ideas that you learned.

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Midterm format

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Midterm format

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Remember that a problem from hw and/or discussions is in the midterm! (identical or almost identical)

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So study those!

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  Also, big proofs are usually a bunch of little proofs put together. And every proof is a new trick. And we like tricks!

Wrapup.

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