CS70: Discrete Math and Probability

Slides adopted from Satish Rao, CS70 Spring 2016

June 20, 2016

Introduction

Programming Computers

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Programming Computers $\equiv$ Superpower!

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What are your super powerful programs doing?

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Logic and Proofs!

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What are your super powerful programs doing?

Logic and Proofs!

Induction $\equiv$ Recursion.

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What can computers do?

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Work with discrete objects.

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Discrete Math

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E.g. machine learning, data analysis.

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Probability!

See note 1, for more discussion.

Admin.

Course Webpage: **www.eecs70.org**

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Explains policies, has homework/discussion worksheets, slides, exam dates, etc.

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Assessment:
Homework: 20%
Midterm 1 (07/08): 20%
Midterm 2 (07/29): 20%
Final (08/12): 35%
Quiz: 4%
Sundry: 1%

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Conflicts? Piazza pinned post.

Learning tips

Take homework seriously!

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Questions?

Staffs

3 Co-Instructors

Fan Ye

Just graduated,

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Been TA for CS70 for two semesters

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Grace Dinh

Email: `dinh@berkeley.edu`
Office Hours: M/W 3:30-5:00 (right after lecture) in 606 Soda
I just finished my first year of grad school. My research interests are numerical algorithms and complexity theory - essentially, I work on making faster algorithms for doing things like solving equations, factoring matrices, etc. (and proving that they run fast!), as well as showing that there are limits on how fast we can make these algorithms.
Also did my undergrad here at Cal - CS70 was by far my favorite lower-div.
Fun fact: I like to make ice cream.

Alex Psomas

Not here today.

Alex Psomas

Not here today. Tomorrow lecture

Staffs

3 Co-Instructors
12 awesome and talented TAs.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

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Suppose we have four cards on a table:

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  "If a person travels to Chicago, he/she flies."

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- Consider the theory:
  "If a person travels to Chicago, he/she flies."
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Alice	Bob	Charlie	Donna
Baltimore	drove	Chicago	flew

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Answer:

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Answer: Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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Today: Note 1.   Note 0 is background. Do read/skim it.

The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational

2+2 = 4

2+2 = 3

826th digit of pi is 4

Stephen Curry is a good player.

All evens >2 are sums of 2 primes

$4+5$

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Propositional Forms.

Put propositions together to make another...

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Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"

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Put them together..

Propositions:

C_1 - Take class 1

Put them together..

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C_1 - Take class 1
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You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

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Propositions:

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Propositional Form:

Put them together..

Propositions:

C_1 - Take class 1
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You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

$$((C_1 \vee C_2) \wedge (C_3 \vee C_4)) \vee ((C_2 \wedge C_3) \wedge (C_5 \vee C_6) \wedge (\neg C_4))$$

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Can you take class 1?
Can you take class 1 and class 5 together?

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We can program!!!!

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We can program!!!!We need a way to keep track!

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
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Notice: $\wedge$ and $\vee$ are commutative.

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One use for truth tables: Logical Equivalence of propositional forms!

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Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$

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DeMorgan's Law's for Negation: distribute and flip!
$\neg(P \wedge Q)$

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$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q$ $\qquad$ $\neg(P \vee Q) \quad \equiv \quad \neg P \wedge \neg Q$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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Simplify: $(T \wedge Q) \equiv Q$,

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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Foil 1:

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$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

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P is True .

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Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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True Statements: P, $P \implies Q$.

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Statement: If you stand in the rain, then you'll get wet.

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Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

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$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

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  Remember if P is true then Q must be true.
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- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

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$\neg P \vee Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
  (Logically Equivalent: $\iff$.)

Variables.

Propositions?

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Next: Statements about boolean valued functions!!

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- See note 0 for more!

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So $P(Bob)$ must be False .

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So $P(Bob)$ must be False .
$P(C)$ = True .

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Only have to turn over cards for Bob and Charlie.

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Note that we may omit universe if clear from context.

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Talk logically

Theorem: No three positive integers a, b, c satisfy the equation $a^n + b^n = c^b$ for any integer n strictly larger than two.

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Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

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Next Time: proofs!