# CS 70 Discrete Mathematics and Probability Theory
# Summer 2016 Dinh, Psomas, and Ye Discussion 5A

**Definition: A random variable $X$ on a sample space $\Omega$ is a function that assigns to each sample point $\omega \in \Omega$ a real number $X(\omega)$**

Until further notice, we'll restrict out attention to random variables that are discrete, i.e., they take values in a range that is finite or countably infinite. Note that the term "random variable" is really something of a misnomer: it is a function so there is nothing random about it and it is definitely not a variable! What is random is which sample point of the experiment is realized and hence the value that the random variable maps the sample point to.

**Definition: The distribution of a discrete random variable $X$ is the collection of values $\{(a, Pr[X = a]) : a \in A\}$, where $A$ is the set of all possible values taken by $X$.**

**1.** Binary Fun

a) What is the sample space $\Omega$ generated by flipping two quarters ($H = 1, T = 0$)? For example, $(H,T) = (1,0)$.

b) Define a random variable $X$ to be the number of heads. What is the distribution of $X$?

c) Define a second random variable $Y$ to be 1 if $\omega = (1,0)$ and 0 otherwise. What is the distribution of $Y$?

d) Define a third random variable $Z = X + Y$. What is the distribution of $Z$?

**2.** Locked Out

You just rented a large house and the realtor gave you five keys, one for the front door and the other four for each of the four side and back doors of the house. Unfortunately, all keys look identical, so to open the front door, you are forced to try them at random.

Find the distribution and the expectation of the number of trials you will need to open the front door. (Assume that you can mark a key after you've tried opening the front door with it and it doesn't work.)

**3.** A roll of the dice

Consider a single roll of two dice, one red and one blue.

1. Let $R$ be the value of the red die. What is the distribution of $R$? What is the expectation of $R$?

2. Let $M$ be the maximum of the numbers on the two dice. What is the distribution of $M$? What is the expectation of $M$?

3. How do the distribution and expectation of $M$ compare to that of $R$?