## CS 70 Discrete Mathematics and Probability Theory Summer 2016 Dinh, Psomas, and Ye Discussion 4B

## 1. Birthdays

Suppose you record the birthdays of a large group of people, one at a time until you have found a match, i.e., a birthday that has already been recorded. (Assume there are 365 days in a year.)

- 1. What is the probability that the first 3 people do not have the same birthday?
- 2. What is the probability that the first three people have the same birthday?
- 3. What is the probability that it takes more than 20 people for this to occur?
- 4. What is the probability that it takes exactly 20 people for this to occur?
- 5. Suppose instead that you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur?

## 2. Balls and Bins

You have n empty bins and you throw balls into them one by one randomly. A collision is when a ball is thrown into a bin which already has another ball.

- 1. What is the probability that the first ball thrown will cause the first collision?
- 2. What is the probability that the second ball thrown will cause the first collision?
- 3. What is the probability that, given the first two balls are not in collision, the third ball thrown will cause the first collision?

- 4. What is the probability that the third ball thrown will cause the first collision?
- 5. What is the probability that, given the first m-1 balls are not in collision, the  $m^{th}$  ball thrown will cause the first collision?
- 6. What is the probability that the  $m^{th}$  ball thrown will cause the first collision?

## 3. Best choice problem

Three princes are going to stop by Alice's house and invite her to their parties. Alice has not met them before so she does not know who she likes most. There are three strategies:

- 1 Go with the first prince.
- 2 Reject the first prince. If she finds she likes the second prince more than the first one, she will go with him. Otherwise, she will go with the third prince.
- 3 Reject the first prince. If she finds she likes the second prince more than the third one, she will go with him. Otherwise, she will go with the third prince.

Which strategy gives Alice the highest probability to go with the prince she likes the most?