## CS 70 Discrete Mathematics and Probability Theory Summer 2016 Dinh, Psomas, and Ye Discussion 4A

## **1.** Counting and Probability Practice

- 1. A message source *M* of a digital communication system outputs a word of length 8 characters, with the characters drawn from the ternary alphabet  $\{0, 1, 2\}$ , and all such words are equally probable. What is the probability that *M* produces a word that looks like a byte (*i.e.*, no appearance of '2')?
- 2. If five numbers are selected at random from the set {1,2,3,...,20}, what is the probability that their minimum is larger than 5? (A number can be chosen more than once.)
- 3. If we put 5 math, 6 biology, 8 engineering, and 3 physics books on a bookshelf at random, what is the probability that all the math books are together?
- **2.** Balls in Bins: Independent? You have *k* balls and *n* bins labelled 1, 2, ..., n, where  $n \ge 2$ . You drop each ball uniformly at random into the bins.
  - 1. What is the probability that bin *n* is empty?
  - 2. What is the probability that bin 1 is non-empty?
  - 3. What is the probability that both bin 1 and bin *n* are empty?
  - 4. What is the probability that bin 1 is non-empty and bin *n* is empty?
  - 5. What is the probability that bin 1 is non-empty given that bin *n* is empty?

## 3. Communication network

In the communication network shown below, link failures are independent, and each link has a probability of failure of p. Consider the physical situation before you write anything. A can communicate with B as long as they are connected by at least one path which contains only in-service links.



- 1. Given that exactly five links have failed, determine the probability that *A* can still communicate with *B*.
- 2. Given that exactly five links have failed, determine the probability that either g or h (*but not both*) is still operating properly.
- 3. Given that *a*, *d* and *h* have failed (but no information about the information of the other links), determine the probability that *A* can communicate with *B*.