CS 70 Discrete Mathematics and Probability Theory
Summer 2016 Dinh, Psomas, and Ye Discussion 3B

1. Manhattan is well-known for its grid layout and busy traffic, so you are interested in evaluating different shortest paths from one point to another.

1. Smile at your neighbor (:
2. Consider two locations that are 3 blocks by 2 blocks away from each other. If you take a cab from one point to the other, how many shortest paths are there?

3. How many shortest paths are there connecting two points that are x blocks by y blocks away from each other? (x and y are non-negative integers.)

4. Assume that an intersection, a blocks by b blocks away from the starting point, is under road work and is not usable. How many shortest paths are there connecting the starting point and the destination

without using that intersection (which is assumed to be between the starting point and the destination, see figure above)?

2. How many solutions does $x_1 + \ldots + x_k = n$ have, if we have the additional constraint that $x_i \geq a_i$, with $a_i \in \mathbf{N}$, for $1 \leq i \leq k$?

3. Prove the following using combinatorial arguments.

1. $\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{k} \cdot \binom{n}{r-k}$

2. $k\binom{n}{k} = n\binom{n-1}{k-1}$