CS 70 Discrete Mathematics and Probability Theory
Summer 2016 Dinh, Psomas, and Ye Discussion 2D

1. Stable Marriage

Consider the following list of preferences:

Men	Preferences
A	$4 > 2 > 1 > 3$
B	$2 > 4 > 3 > 1$
C	$4 > 3 > 1 > 2$
D	$3 > 1 > 4 > 2$

Women	Preferences
1	$A > D > B > C$
2	$D > C > A > B$
3	$C > D > B > A$
4	$B > C > A > D$

1. Is $\{(A,4),(B,2),(C,1),(D,3)\}$ a stable pairing?

2. Find a stable matching by running the Traditional Propose & Reject algorithm.

3. Show that there exist a stable matching where women 1 is matched to men A.

2. Objective Preferences Imagine that in the context of stable marriage all men have the same preference list. That is to say there is a global ranking of women, and men's preferences are directly determined by that ranking.

1. Prove that the first woman in the ranking has to be paired with her first choice in any stable pairing.

2. Prove that the second woman has to be paired with her first choice if that choice is not the same as the first woman's first choice. Otherwise she has to be paired with her second choice.

3. Continuing this way, assume that we have determined the pairs for the first $k-1$ women in the ranking. Who should the k-th woman be paired with?

4. Prove that there is a unique stable pairing.

3. Examples or It's Impossible

Determine if each of the situations below is possible with the traditional propose-and-reject algorithm. If so, give an example of size $n \geq 3$. Otherwise, explain briefly why you think it's impossible.

1. Every man gets his first choice.

2. Every woman gets her first choice, even though her first choice does not prefer her the most.

3. Every woman gets her last choice.

4. Every man gets his last choice.

5. A man who is second on every woman's list gets his last choice.

4. Pairing Up

Prove that for every even $n \geq 2$, there exists an instance of the stable marriage problem with n men and n women such that the instance has at least $2^{n/2}$ distinct stable matchings.