

**1. Leaves in a tree**

A *leaf* in a tree is a vertex with degree 1.

- (a) Prove that every tree on  $n \geq 2$  vertices has at least two leaves.
- (b) What is the maximum number of leaves in a tree with  $n \geq 3$  vertices?

**2. Edge-disjoint paths in hypercube**

Prove that between any two distinct vertices  $x, y$  in the  $n$ -dimensional hypercube graph, there are at least  $n$  edge-disjoint paths from  $x$  to  $y$  (i.e., no two paths share an edge, though they may share vertices).

3. (Odd degree vertices)

**Claim:** Let  $G = (V, E)$  be an undirected graph. The number of vertices of  $G$  that have odd degree is even.

Prove the claim above using:

- (i) Direct proof (e.g., counting the number of edges in  $G$ )
- (ii) Induction on  $m = |E|$  (number of edges)
- (iii) Induction on  $n = |V|$  (number of vertices)
- (iv) Well-ordering principle