

1. (Induction) Prove that, for any positive integer n , $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$.

2. Dividing n -gon

Assume that any simple (but not necessarily convex) n -gon ($n > 3$) has a diagonal (line between two non-adjacent vertices) that lies completely within the n -gon. Show that any such n -gon ($n \geq 3$) can be divided into $n - 2$ triangles such that all vertices of each triangle are vertices of the n -gon.

3. Convergence of Series

Use induction to prove that for all integers $n \geq 1$,

$$\sum_{k=1}^n \frac{1}{3k^{3/2}} \leq 2.$$

Hint: Strengthen the induction hypothesis to $\sum_{k=1}^n \frac{1}{3k^{3/2}} \leq 2 - \frac{1}{\sqrt{n}}$.