# CS 70 Discrete Mathematics and Probability Theory Summer 2016 Dinh, Psomas, and Ye Discussion 1B

## 1. Logic

Decide whether each of the following is true or false and justify your answer: a)  $\forall x (P(x) \land Q(x)) \equiv \forall x P(x) \land \forall x Q(x)$ 

b)  $\forall x (P(x) \lor Q(x)) \equiv \forall x P(x) \lor \forall x Q(x)$ 

## 2. (Proof)

A *perfect square* is an integer *n* of the form  $n = m^2$  for some integer *m*. Prove that every odd perfect square is of the form 8k + 1 for some integer *k*.

### 3. Contradiction

Prove that  $2^{1/n}$  is not rational for any integer n > 3. [Hint : Fermat's Last Theorem ]

### 4. Problem solving

Prove that if you put n + 1 apples into n boxes, any way you like, then at least one box must contain at least 2 apples. This is known as the *pigeonhole principle*.

#### 5. Numbers of Friends

Prove that if there are  $n \ge 2$  people at a party, then at least 2 of them have the same number of friends at the party.